# Irreducible Bases and the Geometry of Prime Configurations

*Critical Alignment, Transported Obstructions, and Parity-Exit Mechanisms*

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**Abstract**

This note is a roadmap to the mathematical development of a multiplicative-geometric corpus for prime configurations. The starting point is the idea that irreducible factors should be treated as geometric directions: states carry logarithmic height, differences have quotient coordinates, and arithmetic constraints can be read through transport. Prime specialization fixes $\lambda_p = \log p$; integers become states, $Z(s) = \sum_{n\geq 1} n^{-s} = \zeta(s)$, and $w = \log(n/m)$ becomes the visible coordinate of multiplicative motion.

Across the corpus the primary object is not a counting sum alone, but a native/observed obstruction class

$$\Delta_{\Phi,X} = \mu^{\mathrm{nat}}_{\Phi,X} - \mu^{\mathrm{obs}}_{\Phi,X}$$

transported to a same-channel terminal readout. The note follows how this grammar organizes critical alignment, Goldbach and its effective closure, fixed-gap fibres, exponential survivor problems, and affine prime channels. A central structural theme is the parity barrier: the accompanying manuscripts propose several distinct parity-exit mechanisms–anti-recycling on additive packets, primitive-zero same-channel transfer for twins, common-fibre native/observed comparison for Sophie Germain, and stopped survivor transfer for Mersenne. The purpose is to make the journey and the resulting change of viewpoint explicit, while leaving technical verification to the individual manuscripts.

Contents

# 1 Purpose and scope

This document gives a historical and conceptual account of the construction of a multiplicative geometric corpus for prime problems. It is best read as a roadmap or background note to the individual technical manuscripts, not as their substitute. Each individual manuscript contains its own definitions, lemmas, estimates, terminal certificates, and local closure arguments. The present note explains how those manuscripts arose, what each one contributed, and how the global architecture changed during the development.

The central sequence documented here is

irreducible bases → prime specialization → internal RH stage
→ Goldbach → effective Goldbach → twin primes
→ Mersenne → Sophie Germain.

The note uses the word *result* in a manuscript-stage sense: a claim stated as a theorem, main conclusion, or corpus-stage construction inside the corresponding manuscript-stage preprint. It does not mean journal publication, peer review, acceptance, or independent validation. The dates in the chronological table are the public Zenodo record dates associated with the DOI records supplied for the corpus; they are not private chat dates, PDF compilation dates, or dates of first conception.

The guiding historical question is:

How did an abstract geometry of irreducible bases become a prime-specialized transport paradigm for several major prime configurations?

# 2 Chronological development of the corpus

The chronology belongs in the body of the preprint because the chronology is not external documentation. It is the backbone of the story. The corpus did not begin as a list of unrelated conjectures. It developed by facing increasingly rigid arithmetic configurations and by converting each new obstruction into a structural tool.

Table 1: Zenodo-dated development ledger of the corpus

| **Zenodo date** | **Corpus stage** | **Mathematical role** |
|---|---|---|
| 13 March 2026 | Irreducible-basis critical model | Abstract multiplicative geometry generated by irreducible directions $\lambda_i$, logarithmic height, difference spectrum, and internal critical-line alignment [1]. |
| 2 April 2026 | Prime-specialized critical model / RH stage | Specialization $\lambda_p = \log p$, arithmetic realization by $\mathbb{N}_{\geq 1}$, quotient coordinate $w = \log(n/m)$, shellwise transport, and zero-localization mechanism for the zeta critical line [2]. |
| 15 April 2026 | Asymptotic binary Goldbach | First arithmetic readout: positivity of the weighted primitive Goldbach function for every sufficiently large even integer [3]. |
| 15 April 2026 | Effective binary Goldbach threshold | Explicit threshold closure for Goldbach, combined with the finite verification range quoted in the threshold manuscript [4]. |
| 31 May 2026 | Twin primes | First fixed-shift endpointal test on the fibre $Q_n = (n, n+2)$ and the terminal identity $x^2 - 1 = n(n+2)$ [5]. |

| Zenodo date | Corpus stage | Mathematical role |
|---|---|---|
| 4 June 2026 | Mersenne primes | Stopped survivor-transfer mechanism on the exponential fibre $2^q - 1$, with terminal divisor trace $\ell = 2kq + 1$ [6]. |
| 18 June 2026 | Sophie Germain primes | Affine rigid-prime channel on the frozen-two carrier $x = 2n$, $Q_x = (x, x + 1)$, with observed event $x/2 \in \mathbb{P}$ and $x + 1 \in \mathbb{P}$ [7]. |

Thus the Zenodo-dated development can be summarized as

13 March 2026: irreducible-basis critical geometry
→ 2 April 2026: prime specialization and RH stage
→ 15 April 2026: Goldbach asymptotic and effective threshold
→ 31 May 2026: twin primes
→ 4 June 2026: Mersenne
→ 18 June 2026: Sophie Germain.

The order is important. The corpus first created a geometry, then specialized it to primes, then read a critical-line mechanism, then forced arithmetic readouts, then confronted rigid channels, then introduced stopped survivor transfer, and finally consolidated the frozen-two affine channel.

## 3 The mathematical spine of the corpus

For an experienced mathematician, the historical sequence is clearer if the common algebraic skeleton is made explicit. The corpus begins with a free multiplicative state space generated by irreducible bases. If $\mathcal{B}$ denotes the set of abstract irreducible bases, a state is a finitely supported exponent vector

$$\alpha = (\alpha_b)_{b\in\mathcal{B}} \in \mathbb{N}^{(\mathcal{B})}, \qquad x(\alpha) = \prod_{b\in\mathcal{B}} b^{\alpha_b}.$$

The geometric height is the logarithmic functional

$$L(\alpha) = \langle \alpha, \lambda \rangle = \sum_{b\in\mathcal{B}} \alpha_b \lambda_b,$$

and the basic quotient displacement between two states is

$$Q(\alpha, \beta) = \alpha - \beta, \qquad w(\alpha, \beta) = L(\alpha) - L(\beta) = \langle \alpha - \beta, \lambda \rangle.$$

Thus multiplication, division, and comparison are all represented by displacement in the same irreducible-coordinate space.

After prime specialization, $\mathcal{B} = \mathbb{P}$ and $\lambda_p = \log p$, so that

$$x(\alpha) = \prod_{p\in\mathbb{P}} p^{\alpha_p}, \qquad L(\alpha) = \log x(\alpha).$$

The Euler product becomes the partition function of the prime-direction geometry:

$$Z_{\mathbb{P}}(s) = \sum_{\alpha\in\mathbb{N}^{(\mathbb{P})}} e^{-sL(\alpha)} = \sum_{n\geq 1} n^{-s} = \prod_{p\in\mathbb{P}} (1 - p^{-s})^{-1}.$$

In this language, the technical branches of the corpus are not separate inventions. They are different terminal readouts of the same transport scheme.

A schematic transport decomposition has the form

$$\mu_X = \mu_X^{\mathrm{vis}} + \mu_X^{\mathrm{hid}} = \mu_X^{\mathrm{al}} + \mu_X^{\mathrm{non}} + R_X,$$

where $\mu_X$ is the geometric mass at scale $X$, the superscripts separate visible/hidden and aligned/nonaligned components, and $R_X$ is the residual mode to be routed or cancelled.

For a prime configuration $\Phi$, the corpus reads arithmetic through a terminal functional

$$\mathcal{R}_\Phi(X) := \langle T_{\Phi,X}\mu_X, \psi_{\Phi,X}\rangle,$$

where $T_{\Phi,X}$ is the configuration-specific transport and $\psi_{\Phi,X}$ is the terminal test function. The general closure pattern is

$$\mathcal{R}_\Phi(X) = M_\Phi(X) + E_\Phi^{\mathrm{vis}}(X) + E_\Phi^{\mathrm{hid}}(X) + E_\Phi^{\mathrm{res}}(X),$$

with the desired terminal positivity or survivor contradiction obtained from

$$M_\Phi(X) > |E_\Phi^{\mathrm{vis}}(X)| + |E_\Phi^{\mathrm{hid}}(X)| + |E_\Phi^{\mathrm{res}}(X)|.$$

This abstract inequality is not meant to replace the local technical proofs. It is the common grammar explaining why RH, Goldbach, twins, Sophie Germain, and Mersenne belong to the same corpus.

## 4 What is mathematically new?

The corpus is most naturally read as a change in the primary object. In the usual analytic formulation, a prime problem is first presented as a counting or correlation problem. For example one starts from objects such as

$$\sum_n \Lambda(n)\Lambda(N-n), \qquad \sum_n \Lambda(n)\Lambda(n+2), \qquad \sum_q \mathbf{1}_{\{2^q-1\in\mathbb{P}\}}.$$

The geometric corpus does not discard these expressions; instead, it treats them as final readouts of a preceding transport problem. The primary object becomes a transported obstruction class.

For a configuration $\Phi$ at scale $X$, the corpus separates a native geometric prediction from the observed arithmetic ledger:

$$\mu_{\Phi,X}^{\mathrm{nat}}, \qquad \mu_{\Phi,X}^{\mathrm{obs}}, \qquad \Delta_{\Phi,X} := \mu_{\Phi,X}^{\mathrm{nat}} - \mu_{\Phi,X}^{\mathrm{obs}}.$$

The observed count is recovered only after applying a terminal readout. In schematic form,

$$C_\Phi(X) = \langle \mu_{\Phi,X}^{\mathrm{obs}}, 1\rangle, \qquad \Omega_{\sigma,\Phi,X}(\tau) = \langle T_{\Phi,X}\Delta_{\Phi,X}, \Psi_{\sigma,\Phi,X}(\tau)\rangle.$$

The insistence on the same $\sigma$, the same transferred class, and the same terminal coordinate is one of the technical firewalls of the corpus. Without it, a zero-pinned or Stokes-type cancellation could occur in a channel unrelated to the arithmetic obstruction.

The contradiction template is then:

$$C_\Phi(X) = 0$$

$$\Downarrow$$

$\Delta_{\Phi,X}$ has macroscopic retained mass

$$\Downarrow$$

$T_{\Phi,X}\Delta_{\Phi,X}$ gives a terminal lower bound

$$\Downarrow$$

same-channel zero-pinning/Stokes gives a terminal upper bound

$$\Downarrow$$

contradiction.

This is the point at which the geometric language becomes mathematically substantive: the target is not just to estimate a sum, but to show that a presumed zero observed ledger produces an obstruction class which cannot survive the same terminal readout that created it.

### Classical object versus geometric replacement

The following table is the expert-facing map of the corpus. It explains, for each branch, what is being replaced by the transport-geometric formulation.

| **Branch** | **Classical starting object** | **Geometric replacement** |
|---|---|---|
| Critical-line stage | Zeros of $\zeta(s)$ and analytic continuation of the Euler product. | Prime quotient geometry with $w = \log(n/m)$, aligned/nonaligned packets, and zero-pinned transfer. |
| Goldbach | Weighted convolution $\sum_h \Lambda(A-h)\Lambda(A+h)$. | A prediction-observation discrepancy on Goldbach packet states, transported to a same-channel terminal readout. |
| Effective Goldbach | Explicit dominance of a major term over all errors. | A frozen parameter block plus threshold ledger in which each downstream error has a certified range. |
| Twin primes | Fixed-shift correlation $\sum_n \Lambda(n)\Lambda(n+2)$. | Endpointal fibre $Q_n = (n, n+2)$ and terminal trace $x^2 - 1 = n(n+2)$ in a finite residual complex. |
| Mersenne | Primality of $2^q - 1$ for prime $q$. | Terminal divisor rows $\ell = 2kq + 1$ and a stopped survivor identity with observed no-hit prefix and promoted future tail. |
| Sophie Germain | Affine prime pair $p, 2p+1$. | Frozen-two carrier $x = 2n$, $Q_x = (x, x+1)$, and same-carrier native/observed ledger comparison. |

Thus the new element is not a single estimate. It is the repeated passage

$$\begin{aligned} &\text{configuration} \\ &\longmapsto \text{ native geometric ledger} \\ &\longmapsto \text{ observed arithmetic ledger} \\ &\longmapsto \text{ retained discrepancy} \\ &\longmapsto \text{ terminal obstruction/readout.} \end{aligned}$$

### Increasing rigidity

The order of the corpus is also an order of increasing rigidity. Goldbach has additive width; the number of possible pairs on the fibre is large. Twin primes remove that width and force a fixed-shift endpointal channel. Mersenne moves to a sparse exponential fibre where the obstruction is not a prime pair but a terminal divisor trace. Sophie Germain then returns to a two-prime condition but on a frozen affine carrier whose native and observed ledgers must live on the same support.

In symbols, the progression is

$$\begin{aligned} \text{Goldbach} &: A-h,\ A+h, \\ \text{twins} &: n,\ n+2, \\ \text{Mersenne} &: q,\ 2kq+1,\ 2^q \equiv 1 \pmod{2kq+1}, \\ \text{Sophie Germain} &: x/2,\ x+1, \quad x \equiv 2 \pmod 4. \end{aligned}$$

The same vocabulary - transport, discrepancy, retained class, readout, terminal obstruction - survives these changes. That survival is the main reason the corpus can be presented as a single program rather than as a collection of unrelated manuscripts.

# 5 The parity barrier and proposed exits in the corpus

One natural way to read the corpus is through the classical parity barrier. In sieve-theoretic language, parity-type obstructions arise because local congruence information is too coarse to distinguish configurations containing the desired prime events from configurations containing products with the same local signature. The corpus does not claim to overcome this obstruction by a single stronger sieve inequality. Instead, the accompanying manuscripts propose to displace parity-type obstructions to several different terminal mechanisms, depending on the fibre.

The common pattern is

$$\text{local survivor information} \not\Rightarrow \text{observed primality},$$

so the corpus never treats local survival itself as the observed prime event. The replacement is

$$\text{zero observed ledger} \Rightarrow \Delta_{\Phi,X} \neq 0 \Rightarrow \text{retained terminal obstruction} \Rightarrow \text{same-channel contradiction}.$$

Thus the proposed parity-exit route is not to declare survivors to be primes. It is to make the failure of observed primality create a macroscopic discrepancy in a channel that can be transported and read terminally.

### Goldbach: tube-residue refinement and anti-recycling

In the Goldbach branch, the parity-type obstruction appears in the nonaligned native-geometric packet mass on the additive fibre. The Goldbach manuscript does not claim that a local sieve ledger directly detects the singular Mangoldt or prime-prime ledger. Instead, it proves that nonaligned mass cannot be recycled indefinitely through logarithmically many comparison levels.

The mechanism is

$$\begin{gathered} \text{tube-residue mismatch} \\ \Downarrow \\ \text{refined nonconcentration} \\ \Downarrow \\ \text{anti-recycling rank and no-return} \\ \Downarrow \\ M_{\mathrm{na}}^{\mathrm{geo},G}(N) \ll N^{1-\theta}. \end{gathered}$$

This is the first proposed parity-exit mechanism of the corpus: the nonaligned component is not asked to distinguish primes from almost-primes directly; it is forced into power saving because it cannot re-enter the transport system through fresh tube labels at many levels. This is the role of the anti-recycling upgrade in the Goldbach manuscript [3].

### Twin primes: primitive-zero discrepancy rather than local primality detection

For twin primes, the barrier is sharper because the additive width has disappeared. The fibre is

$$Q_n^{\mathrm{tw}} = (n, n+2), \qquad x = n+1, \qquad x^2 - 1 = n(n+2).$$

The corpus converts the fixed gap into a terminal factorization trace. The crucial point is negative: the twin-prime manuscript does not use a principle of the form

$$\text{locally primitive} \Rightarrow \text{observed prime-prime}.$$

Observed primality enters only through the observed ledger. Under the zero-observed alternative, the prediction-observation discrepancy creates a primitive-zero residual class, and the same terminal readout family must both detect and annihilate that class. Schematically,

$$T_X = 0 \Rightarrow \Delta_X^{\mathrm{tw}} \neq 0 \Rightarrow \Pi_X^{\mathrm{surv}} \Delta_X^{\mathrm{tw}} \neq 0 \Rightarrow \text{same-channel residual contradiction}.$$

This is a second proposed parity-exit mechanism: the proof avoids identifying local primitivity with twin primality and instead uses the zero observed ledger as the source of the transported obstruction [5].

### Sophie Germain: common affine fibre for native and observed ledgers

The Sophie Germain branch proposes a third parity-exit mechanism. The affine condition is not treated merely as the pair $(p, 2p+1)$. It is placed on the frozen-two carrier

$$x = 2n, \qquad Q_x^{\mathrm{SG}} = (x, x+1), \qquad 2X < x \leq 4X, \qquad x \equiv 2 \pmod 4,$$

with observed event

$$x/2 \in \mathbb{P}, \qquad x+1 \in \mathbb{P}.$$

The native object is a CRT/local-density ledger on this fixed integer carrier; it is not an observed counting measure. The observed Sophie ledger and the native ledger are then realized on the same affine fibre and pushed to the same retained ledger space. The contradiction acts on

$$\Delta_X^{\mathrm{SG}} = \mu_X^{\mathrm{SG,nat}} - \mu_X^{\mathrm{SG,obs}},$$

not on a sieve survivor count identified directly with Sophie Germain primes.

The parity-sensitive step is therefore moved to a common-fibre comparison:

$$S_X^{\mathrm{SG}} = 0$$
$$\Downarrow$$
$$\Delta_X^{\mathrm{SG}} \text{ has retained mass } \gg X$$
$$\Downarrow$$
$$\text{same-channel terminal class}$$
$$\Downarrow$$
$$\text{fixed zero-pinned row family upper bound}.$$

This mechanism is distinct from the twin-prime exit: it uses a frozen affine carrier and a native/observed comparison on the same retained support [7].

### Mersenne: stopped survivor transfer

The Mersenne branch gives the most visibly different proposed parity-exit mechanism. For a prime exponent $q$, every proper prime divisor of

$$M_q = 2^q - 1$$

has the terminal form

$$\ell = 2kq + 1.$$

The divisor obstruction is encoded by the terminal trace

$$D(q,k) = \mathbf{1}_{\{2kq+1 \in \mathbb{P},\ 2^q \equiv 1 \pmod{2kq+1}\}}.$$

Here the proof is not a direct survivor count. It is stopped at the first active terminal row. The stopped identity has the form

$$\left(\prod_{i<j}(1-D_i)\right)(D_j - \lambda_j^\star)\left(\prod_{i>j}(1-\lambda_i^\star)\right).$$

The past is observed, the future is promoted. Dense positive excess becomes genuine double-return structure; sparse positive excess becomes first-hit wall flux; exceptional rows are handled by controlled ledgers. This produces a one-sided observed-promoted Bonferroni transfer rather than a parity-sensitive direct survivor count [6].

### Summary of the proposed parity-exit mechanisms

The accompanying manuscripts therefore propose several structurally different mechanisms for exiting parity-type failure:

| | | |
|---|---|---|
| Goldbach | : | anti-recycling of nonaligned additive packets, |
| Twin primes | : | primitive-zero same-channel residual transfer, |
| Sophie Germain | : | common-fibre native/observed affine discrepancy, |
| Mersenne | : | stopped first-hit survivor transfer. |

This is one of the strongest signals of a change in viewpoint. The corpus does not claim that a single sieve estimate has been sharpened enough to solve several unrelated problems. Rather, the same geometric grammar proposes different terminal exits from parity-type obstructions in different fibres.

## 6 The initial idea: a geometry of irreducible bases

The corpus began with an abstract geometric idea rather than with a particular prime conjecture. Suppose arithmetic objects are expanded over irreducible bases,

$$n = \prod_i b_i^{a_i}.$$

Instead of treating this merely as factorization, the corpus interprets each irreducible base $b_i$ as a geometric direction with logarithmic height $\lambda_i$. The arithmetic object $n$ becomes a state in a multiplicative geometric space.

The basic state diagram is

| irreducible factorization | $\longmapsto$ | geometric state | $\longmapsto$ | height/readout |
|---|---|---|---|---|
| $n = \prod_i b_i^{a_i}$ | | $a(n) = (a_i)_i$ | | $L(n) = \sum_i a_i \lambda_i$. |

Products add exponent vectors, quotients subtract exponent vectors, and logarithmic height turns multiplicative movement into additive geometry:

$$a(nm) = a(n) + a(m), \qquad a(n/m) = a(n) - a(m), \qquad L(nm) = L(n) + L(m).$$

This point of view reverses the usual order of explanation. The irreducible components are not only divisors or factors; they are directions. Products become movements in a geometric lattice, quotients become displacements, and arithmetic configurations become terminal readouts of transport.

The fundamental slogan of this first stage is

| multiplication generates geometry. |
|---|

The basic language introduced at this stage includes irreducible directions, logarithmic height, quotient displacement, packet structure, alignment and nonalignment, visible and hidden mass, residual modes, and transport between local and terminal readouts.

At this stage, the model is not yet a model of primes. It is a general irreducible-basis geometry. Its purpose is to create a language in which arithmetic phenomena can be interpreted as geometric transport phenomena.

## 7 From abstract irreducibles to prime specialization

The decisive specialization is

$$\lambda_i \rightsquigarrow \lambda_p = \log p.$$

The abstract irreducible directions are now the prime directions. The state space becomes the multiplicative space of positive integers,

$$n = \prod_p p^{v_p(n)}.$$

The Euler product

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1}$$

is then not merely an analytic identity but a global partition structure over the prime-direction geometry.

Equivalently,

$$\zeta(s) = \sum_{\alpha \in \mathbb{N}^{(\mathbb{P})}} \exp\big(-s\langle \alpha, (\log p)_p \rangle\big).$$

This formula is the bridge between the classical analytic object and the geometric state space. The prime basis is no longer only the support of an Euler product; it is the coordinate system of the model.

This prime specialization introduces the quotient coordinate

$$w = \log(n/m) = \sum_p \big(v_p(n) - v_p(m)\big) \log p,$$

which becomes one of the basic geometric readout coordinates of the corpus. Once quotients are seen as displacements, arithmetic comparison becomes geometric transport.

The prime-specialized stage contributes prime directions $\lambda_p = \log p$, Eulerian partition structure, quotient coordinates, shellwise transport, aligned and nonaligned packets, visible/hidden decomposition, local packet readouts, residual tails, and anti-recycling mechanisms. This stage is the bridge from abstract geometry to number theory. All later applications depend on this specialization.

## 8 The RH stage: internal critical alignment

The first major test of the prime-specialized model was the critical-line stage. The guiding principle was that the line

$$\Re(s) = \frac{1}{2}$$

should not be imposed as an external analytic boundary. Instead, it should arise from an internal alignment condition of the multiplicative prime geometry.

The RH-stage introduced a difference spectrum attached to multiplicative quotient geometry. At a schematic level, one studies a transported spectral packet

$$\mathcal{Z}_X(s) = \mathcal{Z}_X^{\mathrm{cen}}(s) + \mathcal{Z}_X^{\mathrm{osc}}(s) + \mathcal{Z}_X^{\mathrm{off}}(s),$$

with a zero-pinned condition of the form

$$D^j \mathcal{Z}_X(\rho_0) = 0, \qquad 0 \le j \le m,$$

and transport estimates designed to force the off-critical packet to be negligible at the terminal readout:

$$\|\mathcal{Z}_X^{\mathrm{off}}\|_{\mathrm{terminal}} = o(1).$$

The critical line is then read as the stable locus of the central aligned component:

$$\mathcal{Z}_X^{\mathrm{cen}} \quad \rightsquigarrow \quad \Re(s) = \frac{1}{2}.$$

The conceptual contribution of this stage is that the corpus no longer treats zeros merely as objects located by analytic continuation. Instead, it asks how the critical line is forced by the equilibrium of the prime-direction geometry.

In schematic form:

$$\boxed{\text{prime directions} \to \text{quotient spectrum} \to \text{internal alignment} \to \Re(s) = 1/2.}$$

This stage gave the corpus its first global organizing principle: arithmetic visibility should be read through aligned geometric transport rather than through raw counting alone.

## 9 Goldbach as the first arithmetic readout

Goldbach was the first point where the corpus had to become arithmetically operational. The problem is to read representations

$$2N = p + q.$$

The transition from the RH-stage to Goldbach is therefore the transition from spectral alignment to arithmetic mass production.

Classically one may write the weighted Goldbach correlation as

$$G_\Lambda(2N) = \sum_{n \le 2N} \Lambda(n)\Lambda(2N-n),$$

with the expected main scale

$$G_\Lambda(2N) \sim 2\mathfrak{S}(2N)N.$$

The corpus reformulates this as a terminal readout of transported prime mass:

$$\mathcal{R}_G(N) = \langle T_N^G(\mu_N^{\mathrm{vis}} + \mu_N^{\mathrm{hid}}), \mathbf{1}_{p+q=2N} \rangle.$$

The schematic closure is

$$\mathcal{R}_G(N) = M_G(N) + E_G^{\mathrm{non}}(N) + E_G^{\mathrm{wall}}(N) + E_G^{\mathrm{res}}(N),$$

where the aligned weighted main term dominates the nonaligned, wall-flux, and residual terms:

$$M_G(N) \asymp \mathfrak{S}(2N)N, \qquad E_G^{\mathrm{non}} + E_G^{\mathrm{wall}} + E_G^{\mathrm{res}} = o(M_G).$$

If one later passes from logarithmic weights to an unweighted count, the corresponding heuristic scale is $\mathfrak{S}(2N)N/(\log N)^2$; the manuscript-stage Goldbach readout itself is the weighted positivity statement.

The Goldbach stage introduced or consolidated Goldbach packets, visible/hidden mass transfer, shellwise closure, primitive aligned prediction, residual transport, compensator estimates, wall-flux bounds, Bonferroni transfer, and anti-recycling of residual mass.

Goldbach played a structural role far beyond its own statement. It was the first proof environment in which the prime-specialized geometry was required to output a concrete arithmetic readout. The geometric question became:

| does aligned transport force visible prime-pair mass at the terminal readout? |
|---|

The Goldbach manuscript therefore converted the corpus from a critical-line geometry into a prime-configuration machine.

## 10 Effective Goldbach and the birth of certificates

The asymptotic Goldbach stage was not the end of the Goldbach branch. A second step was required: the effective threshold.

This step changed the standard of the corpus. An asymptotic theorem can be organized around main terms and error terms. An effective threshold requires every constant, bound, dependency, and finite range to be explicitly controlled.

In the effective stage the schematic inequality becomes a certified inequality. For a threshold $N_0$, the target is

$$M_G(N) - \Big(|E_G^{\mathrm{non}}(N)| + |E_G^{\mathrm{wall}}(N)| + |E_G^{\mathrm{res}}(N)|\Big) > 0, \qquad N \geq N_0.$$

The finite bridge has the form

$$4 < N < N_0 \quad \Longrightarrow \quad \text{verified directly or imported through an explicit finite certificate.}$$

Thus a proof is no longer only a chain of estimates. It becomes a tuple

$$\mathfrak{C}_G = (N_0, \mathcal{E}, \mathcal{L}, \mathcal{V}, \mathcal{D}),$$

where $\mathcal{E}$ records explicit estimates, $\mathcal{L}$ records locked dependencies, $\mathcal{V}$ records the finite verification bridge, and $\mathcal{D}$ records the data or source table needed for terminal closure.

The effective Goldbach stage introduced the certificate culture of the corpus: explicit thresholds, finite verification ranges, source ledgers, dependency ledgers, locks, terminal certificates, printed or externally referenced data, and no-pending-certificate discipline.

The historical importance of this stage is not only the threshold itself. Its importance is methodological: it forced the corpus to distinguish between conceptual closure and effective certified closure.

The governing principle became:

| a ledger records a proved dependency; it must not prove that dependency by declaration. |
|---|

This principle later became decisive in the audit of rigid channels and terminal certificates.

## 11 Twin primes: the first endpointal fixed-gap test

The twin-prime stage is a separate branch of the corpus and cannot be hidden inside Sophie Germain or generic fixed-gap language. Its arithmetic configuration is

$$p, \quad p+2.$$

This is fundamentally different from Goldbach. Goldbach has additive width: many pairs may contribute to a fixed even integer. Twin primes impose an endpointal fixed-gap channel. The mass has much less room to move.

The classical weighted correlation is

$$\Pi_{2,\Lambda}(X) = \sum_{n \le X} \Lambda(n)\Lambda(n+2),$$

where the Hardy-Littlewood scale would be

$$\Pi_{2,\Lambda}(X) \sim 2C_2 X.$$

The corpus reinterprets this as a same-channel endpointal readout:

$$\mathcal{R}_{\mathrm{tw}}(X) = \langle T_X^{\mathrm{tw}} \mu_X, \psi_X^{\mathrm{tw}} \rangle, \qquad \psi_X^{\mathrm{tw}}(n) = \mathbf{1}_{\mathbb{P}}(n)\mathbf{1}_{\mathbb{P}}(n+2).$$

The new difficulty is that the residual boundary cannot be averaged away by additive width. It must be controlled on the endpointal channel itself:

$$\left| \langle (\pi_{\mathrm{obs}})_* \partial B_X^{\mathrm{tw}}, c_X \rangle \right| \le C_\varepsilon X^{1/2+\varepsilon}.$$

This is the schematic role of the critical endpointal boundary estimate: it prevents residual boundary mass from recycling into the same terminal channel.

The twin-prime stage introduced the first fully rigid endpointal test of the transport geometry. The key conceptual transition is

| Goldbach tests transport with width; twin primes test transport in a rigid channel. |
|---|

The main tools associated with this stage are fixed-gap fibres, endpointal atlas, endpointal residual transport, critical endpointal boundary estimates, same-channel terminal transport, primitive observed readouts, readout separation, endpointal anti-recycling, and residual-cycle impossibility.

The twin-prime branch gave the corpus its first serious encounter with the rigidity of fixed-gap arithmetic. It also created much of the language later used in the Sophie Germain stage.

## 12 Mersenne: stopped survivor transfer

The Mersenne branch introduced a different kind of arithmetic fibre. For prime exponent $q$, the Mersenne number is

$$M_q = 2^q - 1.$$

Every possible proper prime divisor has the form

$$r = 2kq + 1.$$

This creates a sparse exponential fibre rather than an additive or fixed-gap channel.

A natural terminal divisor trace is

$$D(q,k) = \mathbf{1}_{\mathbb{P}}(2kq+1)\, \mathbf{1}_{\{2^q \equiv 1 \bmod (2kq+1)\}}.$$

A survivor set at scale $Q$ can be schematically written as

$$\mathcal{S}(Q) = \left\{ q \le Q : \ q \in \mathbb{P}, \sum_{k \le K(q)} D(q,k) = 0 \right\}.$$

The Mersenne readout is therefore not a convolutional count but a terminal survivor functional:

$$\mathcal{R}_M(Q) = \sum_{q \le Q} \mathbf{1}_{\mathbb{P}}(q) \prod_{k \le K(q)} (1 - D(q,k)).$$

The stopped survivor-transfer mechanism controls this readout by stopping the transport at the divisor fibre $r = 2kq + 1$:

$$\mu_Q \xrightarrow{T_Q^{\mathrm{stop}}} \{(q,k) : 2kq+1 \text{ is terminally visible}\} \xrightarrow{D} \text{survivor ledger}.$$

The Mersenne stage contributed the stopped survivor-transfer mechanism. Instead of following all global mass transport, the method stops the flow at a terminal fibre and asks which possible divisors survive.

The associated tools are terminal divisor trace, survivor ledger, final visible quotient, stopped identities, one-sided survivor transfer, compensator estimates, low/dense/exceptional decomposition, and terminal survivor contradiction.

The conceptual contribution of Mersenne is captured by

| do not follow the entire flow; stop it at the terminal fibre and control the survivors. |
|---|

This mechanism closes the historical arc treated in this note: from irreducible-basis geometry, through prime specialization and arithmetic transport, to terminal survivor control on a sparse exponential fibre.

## 13 Sophie Germain: an affine rigid-prime channel

The Sophie Germain stage concerns primes $p$ for which $2p+1$ is also prime. In the corpus this is not treated merely as the schematic correlation $\sum \Lambda(n)\Lambda(2n+1)$. The actual carrier is the frozen-two consecutive fibre

$$x = 2n, \qquad Q_X^{\mathrm{SG}}(x) = (x, x+1), \qquad 2X < x \le 4X, \qquad x \equiv 2 \pmod 4.$$

The observed Sophie event is

$$\frac{x}{2} \in \mathbb{P}, \qquad x+1 \in \mathbb{P}.$$

Thus the active odd divisor walls are not written on an abstract affine line alone, but on the two consecutive channels

$$L_0(x) = x, \qquad L_1(x) = x+1, \qquad D_{s,\ell}(x) = \mathbf{1}_{\ell \mid L_s(x)}, \quad s \in \{0,1\},\ \ell > 2.$$

For each odd prime $\ell$, the two forbidden classes are distinct,

$$x \equiv 0 \pmod \ell, \qquad x \equiv -1 \pmod \ell,$$

so the local survivor factor is

$$1 - \frac{2}{\ell}.$$

The observed ledger and the native ledger live on the same frozen carrier:

$$\mu_X^{\mathrm{SG,obs}}(x) = \mathbf{1}_{x/2 \in \mathbb{P}} \mathbf{1}_{x+1 \in \mathbb{P}} \log(x/2) \log(x+1),$$

while $\mu_X^{\mathrm{SG,nat}}$ is the native principal measure obtained from the same archimedean weight and the local survivor factors. The transported discrepancy is

$$\Delta_X^{\mathrm{SG}} = \mu_X^{\mathrm{SG,nat}} - \mu_X^{\mathrm{SG,obs}}.$$

Under the zero-observed alternative, this discrepancy carries the native principal mass; the proof then projects it to the same survivor quotient, applies the same-channel terminal transfer, and tests it against a fixed terminal readout family. The structural lesson is that the Sophie Germain branch is a rigid affine-prime channel, but its final proof grammar is frozen-two and consecutive:

$$p, 2p+1 \quad \Longleftrightarrow \quad x/2, x+1 \quad \text{on } x \equiv 2 \pmod 4.$$

This stage inherited endpointal technology from the twin-prime branch but introduced additional demands. The corpus had to control a rigid affine fibre while also making the manuscript autonomous. A recurring editorial requirement was that the Sophie Germain manuscript should not rely on companion papers or hidden corpus imports.

The Sophie Germain stage emphasized autonomy of the manuscript, avoidance of companion-paper dependency, noncircular ordering of appendix, lemma, and theorem chains, bad-atom routing by first failure, proof before ledger registration, and terminal closure without conditional hypotheses.

A typical dependency concern in this stage had the form

$$\text{Appendix K} \longrightarrow \text{Lemma 3.3} \longrightarrow \text{Theorem 3.5}.$$

The audit question was whether any part of the appendix was implicitly using the final theorem it was supposed to support. This pushed the corpus toward a stricter dependency grammar. In schematic form, bad compression or bad routing atoms had to be assigned to already controlled first-failure families:

$$\mathcal{B}_X^{\mathrm{bad}} \subset \mathcal{F}_X^{(1)} \cup \cdots \cup \mathcal{F}_X^{(r)},$$

where each $\mathcal{F}_X^{(j)}$ is controlled before the final theorem uses the bad-atom decomposition.

The main lesson of the Sophie Germain stage was:

| a ledger can record a terminal class only after the class has been materially realized. |
|---|

## 14 How the tools were born

A mathematician reading the corpus should not see a sequence of disconnected claims. Each stage was forced by a specific new obstruction.

| Stage | New obstruction | Tool or viewpoint created |
|---|---|---|
| Irreducible bases | Arithmetic factorization lacked a geometric state space. | Directions $\lambda_i$, quotient displacement, multiplicative packets. |
| Prime specialization | The abstract geometry had to become arithmetical. | Prime directions $\lambda_p = \log p$, Eulerian partition geometry. |
| RH stage | The critical line had to be internal, not imposed. | Critical alignment, difference spectrum, zero-pinned transfer. |
| Goldbach | Geometry had to produce arithmetic mass. | Visible/hidden transport, shellwise closure, aligned prediction. |
| Effective Goldbach | Asymptotic closure was not enough. | Thresholds, locks, finite verification bridge, certificates. |
| Twin primes | Additive width disappeared. | Endpointal fixed-gap channel, same-channel terminal transport. |
| Sophie Germain | Fixed-gap rigidity became affine rigidity. | Autonomy grammar, bad-atom routing, noncircular closure. |
| Mersenne | The fibre became sparse and exponential. | Stopped survivor transfer, terminal divisor traces, survivor ledgers. |

The repeated pattern was:

$$\boxed{\begin{array}{c}\text{claim} \to \text{technical objection} \to \text{detected dependency gap} \\ \to \text{new lemma/certificate} \to \text{circularity check} \to \text{editorial cleanup.}\end{array}}$$

This method shaped the final corpus. Each serious objection was not treated as a stylistic problem but as evidence that the dependency grammar needed to be improved. The final standard can be summarized as follows:

$$\boxed{\text{nothing enters the terminal contradiction merely because it appears in the ledger.}}$$

## 15 Comparison with traditional approaches

The traditional analytic strategy begins with a function to be estimated. In additive prime problems this is usually a convolution; in fixed-gap problems it is a shifted correlation; in Mersenne-type problems it is a primality or divisor-detection problem. The basic proof shape is

$$\text{arithmetic sum} = \text{main term} + \text{error}, \qquad \text{main term} > |\text{error}|.$$

Circle-method, sieve, dispersion, and zero-density tools differ greatly in detail, but they share this order: define a counting object, estimate it, and try to prove positivity.

The multiplicative-geometric corpus changes this order. It begins from irreducible directions and asks how the observed arithmetic ledger is produced from a native geometric ledger. The central object is not immediately the final count; it is the discrepancy

$$\Delta_{\Phi,X} = \mu^{\mathrm{nat}}_{\Phi,X} - \mu^{\mathrm{obs}}_{\Phi,X}$$

after both sides have been placed on the same retained coordinate system. The terminal readout is then required to be same-channel:

$$\Omega_{\sigma,\Phi,X} = \langle T_{\Phi,X}\Delta_{\Phi,X}, \Psi_{\sigma,\Phi,X}\rangle.$$

This is the structural distinction most visible to an expert reader. The corpus does not merely rename the main term and the error term. It introduces an intermediate geometric object - the retained transported obstruction class - and makes the final contradiction depend on that class being read in exactly the same terminal channel from above and below.

| **Traditional viewpoint** | **Multiplicative geometric viewpoint** |
|---|---|
| Primes are detected by weights, congruence exclusions, or analytic continuation. | Primes are irreducible directions with length $\log p$. |
| The primary object is a sum or correlation. | The primary object is a native/observed ledger pair and its discrepancy. |
| Major/minor arcs or sieve weights separate contributions. | Aligned/nonaligned and visible/hidden transport separate channels. |
| Errors are bounded globally. | Residual modes are routed to retained terminal coordinates. |
| Exceptional sets are discarded or absorbed. | Bad atoms are assigned to first-failure families before terminal closure. |
| The contradiction is numerical positivity. | The contradiction is same-channel incompatibility between a terminal lower bound and a zero-pinned/Stokes upper bound. |

This comparison should be read modestly. The corpus does not claim that classical analytic number theory is irrelevant. Indeed, several stages still use familiar local factors, major-arc language, finite verification, or sieve-like comparisons. The claim is more specific: the explanatory center is moved from the final analytic estimate to the geometric mechanism that produces the terminal readout.

## 16 Paradigm shift

The paradigm shift can be stated without rhetoric:

| prime configurations are treated as terminal manifestations of multiplicative transport. |
|---|

Goldbach is not only a convolution; it is the first arithmetic readout of transported prime-packet mass. Twin primes are not only a shifted correlation; they are the first endpointal fixed-gap stress test. Mersenne primes are not only a search for prime values of $2^q - 1$; they are a stopped survivor problem on the terminal divisor trace $\ell = 2kq + 1$. Sophie Germain primes are not only the affine pair $p, 2p + 1$; they are realized on the frozen-two carrier $x = 2n$, $Q_x = (x, x + 1)$, with native and observed ledgers compared on the same support.

The mature form of the corpus is therefore the following diagram:

$$\begin{array}{c}\text{irreducible directions}\\ \Downarrow\\ \text{prime specialization } (\lambda_p = \log p)\\ \Downarrow\\ \text{quotient geometry } w = \log(n/m)\\ \Downarrow\\ \text{native/observed ledger comparison}\\ \Downarrow\\ \text{retained transported obstruction class}\\ \Downarrow\\ \text{same-channel terminal readout}\\ \Downarrow\\ \text{positivity or survivor contradiction.}\end{array}$$

This is the point of the historical account. The corpus did not grow by adding conjectures one after another. It grew by forcing the same transport grammar through increasingly rigid prime configurations. Each new branch created a new stress test: critical alignment, additive readout, effective certification, endpointal rigidity, stopped survival, and affine frozen-two closure.

The final conceptual message is

| from counting primes as isolated events to reading prime configurations as terminal outputs of irreducible multiplicative geometry. |
|---|

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